

## UNCERTAINTY CHARACTERIZATION IN QUANTITATIVE MODELS

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### ABSTRACT

The treatment of uncertainty quantification has evolved over the years. One of the main drivers of this sweeping paradigm shift has been the advances in computing power. Today, with the exponential advances in computing power relative to the past, we can collect an unprecedented amount of data over an almost unlimited period of time. Big Data, as folks in the statistical analytics field like to call it, has expanded past the barriers of previous limitations of data analytics. But there's a problem; our over-reliance on computer analytics and our insatiable appetite to 'play with the numbers' means more often than not, we cannot spot mistakes within our computer models and blindly follow what the numbers say. As powerful as computers are, and as advanced as analytics platforms have become, quantitative models perform optimally when guided by expert intuition. These computer based models grapple not only with physical variabilities in the real system they emulate, but also with potential deficiencies within the model itself due to a lack of knowledge about the system being modeled or its surrounding environment. Today, every industry seeking a competitive edge is using data to shape its decision making under uncertainty in one way or the other. This paper, following recent advances in uncertainty quantification in the leading domains of engineering, statistics and climate change, presents the current state of the practice within these domains. The paper also presents a synthesis of recent research.

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## 1. BACKGROUND

Model uncertainties reflect the inability of a model or design technique to represent a system's true physical behavior precisely, or the analyst's inability to identify the best model, or a model that may be changing in time in poorly known ways (e.g., a flood-frequency curve changing because of changing watershed conditions). The models used to approximate naturally varying phenomena need to be fit to natural processes by observing how those processes work, by measuring important features, and by statistically estimating parameters of the models within the broad state of knowledge that we have about the processes. In the modeling literature this is sometimes referred to as *uncertainty quantification*.

## 2. NATURE OF UNCERTAINTY

The character and importance of uncertainty in dam safety risk analysis drives how risk assessments are used in practice. The current interpretation of uncertainty is that, in addition to the aleatory risk which arise from presumed uncertainty in the world, it comprises the epistemic aspects of irresolution in a model or forecast, specifically model and parameter uncertainty (Figure 1). This is true in part but it is not all there is to uncertainty in risk analysis. The physics of hazards and of failure may be poorly understood, which goes beyond uncertainty in its conventional sense. All of these facets are part of the uncertainty in risk analysis with which we must deal. From a practical view, one might distinguish three types of uncertainty in risk analyses: variation in nature (aleatory uncertainty), knowledge limitations (epistemic uncertainty), and unknowns (deep uncertainty).

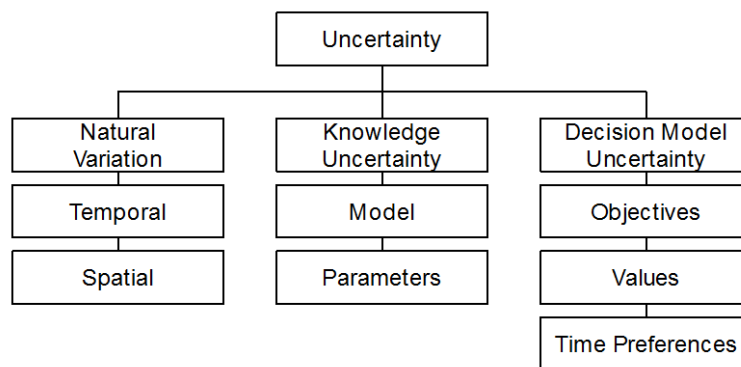


Figure 1. Sources of uncertainty (Baecher and Christian, 2000).

### 2.1. Aleatory uncertainty: Variation in nature

Since antiquity, people have thought of nature and the vagaries of life as uncertain. The world itself is driven by fortune and luck. In modern practice, we treat rainfall, earthquakes, hurricanes, and many other natural hazards as innately random. Their randomness is part of the natural world, irrespective of people and what people know. Were there no people, these natural processes would still go on, and the frequencies with which they occur would be unchanged. The hydrology literature has traditionally adopted this point of

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view. These uncertainties are said to be *aleatory* meaning innately random. In some fields, such as climate change modeling, they are referred to as *ontological* uncertainties.

Aleatory uncertainties are most often natural frequencies in time or space. They are predictable up to a probabilistic description, and their uncertainty can never be reduced below their naturally occurring frequencies, irrespective of how much we may know about them or how many data we may observe.

In applying probability measures to such uncertainties, the meaning of term *probability* is usually taken to be the frequency of occurrence in a long or infinite series of similar trials. In this sense, probability is interpreted for operational matters to be a property of the system (*i.e.*, a property of nature) independent of anyone's knowledge of it or evidence for it. We may or may not know what the value of this probability is, but the probability in question is a property for us to learn. It is innate; there is a "true" value of this probability. Two observers, given the same evidence, and enough of it, should eventually converge to the same numerical value.

### 2.2. Epistemic uncertainty: Limitations in knowledge

Sometime in the 1970's it became increasingly obvious that not only were aleatory uncertainties important, but parameter and model uncertainty were, too. These latter uncertainties had nothing to do with natural variations in time and space, but with information: how complete were the data upon which the model characterizations were based. These were not uncertainties in the world but uncertainties in the mind. They had to do with how much one knew, and they could be reduced essentially to zero by collecting ever greater numbers of data.

The recognition of parameter and model uncertainty as distinct from randomness is important in modern risk analysis. To simplify the task of risk assessment, one makes assumptions about how to grapple with uncertainties. By far the most important of these assumptions is separating uncertainty between aleatory and epistemic, between natural variations over space and time and lack of knowledge in the mind of the analyst or in the broader informed technical community (Table 1).

*Table 1. Alternate terms describing the dual meaning of uncertainty.*

ALEATORY UNCERTAINTY	EPISTEMIC UNCERTAINTY	CITATION
Natural variability	Knowledge uncertainty	(NRC, 2000)
Random or stochastic variation	Functional uncertainty	(Stedinger et al., 1996)
Objective uncertainty	Subjective uncertainty	(Maidment, 1993)
External uncertainty	Internal uncertainty	(Maidment, 1993)
Statistical probability	Inductive probability	(Carnap, 1936)
<i>Chance</i> [Fr]	<i>Probabilité</i> [Fr]	Poisson, Cournot (Hacking, 1975)

The distinction between these two types of uncertainty can have profound impact on risk, and on the meaning that one ascribed to risk. Yet, the questions raised by this fundamental distinction are by no means simple to answer. Most uncertainties are a mixture of things, so how does one practically differentiate natural variation from limited knowledge? Since the

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two types of uncertainty reflect conceptually different things, how does one quantify each? If probability theory is used as a measure of uncertainty, are different types of probability needed for different types of uncertainties? Can and should the two types of uncertainty be combined? If they can and should be combined, how does one do so? These issues are not limited to the analysis of dam safety and flood damage; they are just as important to seismic hazard, structural reliability, wind threat, and other risks of concern to the built environment.

### 2.3. Sources of uncertainty

Uncertainty enters risk analysis models in many ways (Figure 2). Hazards of various types, such as flood loading, seismic ground shaking, SCADA malfunctions, or human error serve as input to the model. These are combined and processed by the model, and consequences are predicted as output. The model itself and a characterization of the hazards also require a variety of parameters. These reflect natural and other conditions, and calibrate the model to reality.

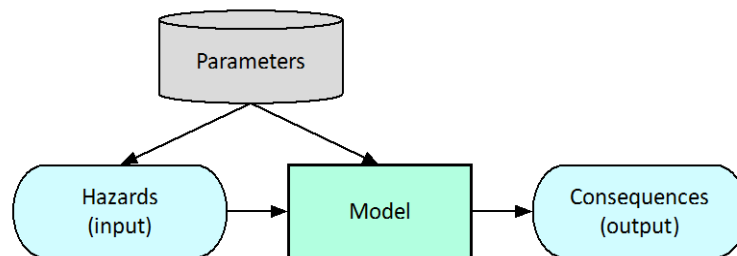


Figure 2. Predictive risk model schematic

**Parameter uncertainty.** The parameter values which serve as input both to the hazard characterization and to the risk model are usually not known precisely. There may be statistical error in estimating these values from historical data, or there may be experimental error in measuring these values in the laboratory or *in situ*. These are sometimes called, Type A evaluations of uncertainties. Knowledge about the parameter values may also come from engineering judgment or other information concerning the quantity. These are sometimes called, Type B evaluations of uncertainties.

**Parameter uncertainties** result from an inability to assess exactly the parametric values from test or calibration data due to limited numbers of observations and the statistical imprecision attendant thereto. These include data uncertainties deriving from (i) measurement errors, (ii) inconsistency of data, (iii) data handling and transcription errors, and (iv) poor representativeness of sampling schemes due to time and space limitations.

**Structural or model uncertainty.** The calculation will model itself and the mathematical equations by which it is expressed may themselves be inadequate or simplifications. Models are almost always approximations of reality based on assumptions which are made for expedience or for calculational convenience. There may also be latent variables in a model which are either ignored or possibly unknown. Thus, discrepancies are always expected between the model and true physics.

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**Algorithmic or numerical uncertainty.** Algorithmic uncertainty arises from numerical errors or numerical approximations made in implementing the model in computer code. Most engineering models in dam safety are too complicated to solve exactly. For example, finite element (*i.e.*, numerical) models may be used to calculate stress or seepage patterns, or event trees may be used to propagate uncertainties through stages of a failure. Linearization, truncation, interpolation, and other approximations may be introduced to make these calculations more efficient. This results in some level of error.

**Initial and boundary condition uncertainty.** The initial and boundary conditions of a model need to be specified before calculations can be made. Since many of the models in dam safety are differential, initial conditions state the value of a function or process at some time, usually  $t=0$ . Similarly, boundary conditions state the value of a function or process at some location or boundary to the calculation.

### 3. UNCERTAINTY PROPOGATION

There are two categories of problems in uncertainty quantification: the *forward* propagation of uncertainty (where the sources of uncertainty are propagated through the model to predict overall uncertainty in the output), and *inverse* assessment where the model parameters are calibrated against known output. This section considers forward propagation.

The general methodology in predictive modeling is to estimate a function that best characterizes the parameters being modeled. More generally, suppose we observe a quantitative response  $Y$  and  $P$  different predictors, a relationship between  $Y$  and  $p$  is assumed, then the general mathematical representation of this relationship is of the form

$$Y = f(X) + \varepsilon \quad (1)$$

Here  $f$  is some fixed but unknown function of  $X_1, \dots, X_p$ , and  $\varepsilon$  is a random error term, which is independent of  $X$  and has mean zero. This general expression  $f$ , models the information provided by the predictors (input) variables about  $Y$  (Output variables).

This is the fundamental basis of predictive modeling with the function  $f$  representing the predictor model that allows us to understand which components of  $X$  are important in explaining  $Y$ . There are several predictive modeling techniques of varying complexities of the function  $f$ . Introducing more complexity (flexibility) into the function  $f$ , may or may not lead to an improved model but certainly reduces the interpretability of the model.

Typically, we have to estimate the form of the function  $f$  in order to be able to make predictions and inferences. For predictions, Figure 1 shows the schematic of generating the response estimates  $Y$  given the predictor variables  $X$ . In this setting, the function  $f_0$  (model) can be treated as a black box since the mathematical form of the function is of little concern to us provided it's generating accurate predictions. The accuracy of the prediction for the response  $Y$  is dependent on two error quantities; the irreducible and reducible error. As expected, the function  $f_0$  will not be a perfect estimate for  $f$  and will introduce some error (reducible error). Statistical learning is therefore concerned with improving this reducible error in order to improve the accuracy of our model predictions.

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The reducible error by definition is an epistemic uncertainty which can be improved through fine tuning our modeling methodology. However, the irreducible error here is an aleatory uncertainty since the irreducible error is characterized by unmeasurable variation. For instance, let's consider a model estimate  $f_o$  with predictor sets of  $X$ ; which then yields  $Y_o = f_o(X)$ . Assuming  $f_o$  and  $Y$  to be fixed, we can deduce the mathematical form for the errors in our predictions.

$$\begin{aligned} E(Y - \hat{Y})^2 &= E[f(X) + \varepsilon - f_o(X)]^2 \\ &= [f(X) - f_o(X)]^2 + Var(\varepsilon) \end{aligned} \quad (2)$$

Where  $E(Y - \hat{Y})^2$  represents the average, or expected value, of the squared difference between the predicted and actual value of  $Y$ , and  $Var(\varepsilon)$  represents the variance associated with the error term  $\varepsilon$ .  $[f(X) - f_o(X)]^2$  is the reducible error resulting from the discrepancy between the model outputs and the true responses of the system.

The irreducible error will always restrain the accuracy of statistical predictions since it's almost always unknown in practice (James et al., 2013). Much of statistical learning is therefore concerned with finding the balance in complexity and accuracy when estimating  $f$ . There are different methods of predicting the output  $Y$  from our model  $f_o$  with a variety of predictive modeling methods adopted throughout the systems model formulation and construction. Broadly speaking, these methods can be classified into two main approaches; parametric methods and non-parametric methods.

The parametric methods make an assumption about the functional form/shape of the function  $f$ . For example in a linear regression analysis the relationship between the predictor  $X$  and the response  $Y$  is assumed to be linear; thus  $f$  is linear. This greatly simplifies the analysis. Linear regression is very useful in lots of applications but has its limitations as things in the real world don't always follow a linear pattern and in such scenarios a simple linear regression function will increase the reducible error and not lead to accurate predictions. The general form of the response function of a simple linear regression is,

$$Y \approx \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_p X_p \quad (3)$$

The linear assumption here means that the task of estimating the function  $f$  boils down to estimating the set of parameters  $(\beta_0, \beta_1, \beta_2, \dots, \beta_p)$ . The higher the flexibility of the parametric model, the more parameters we need to account for in our model. Linear models are the most basic form of parametric modeling. The data are normally governed by some parametric probability distribution. This means that the data can be interpreted by one or other mathematical formula representing a specific statistical probability distribution that belongs to a family of distributions differing from one another only in the values of their parameters.

Such a family of distributions may be grouped accordingly:

- Beta distribution
- Binomial distribution
- Lognormal distribution
- Exponential (Poisson) distribution

- Weibull distribution.

Estimation techniques for determining the level of confidence related to an assessment of reliability based on these probability distributions are the methods of maximum likelihood, and Bayesian estimation (Stapelberg, 2009). A couple of these distributions are adopted throughout the modeling process.

### 3.1. Non Parametric Methods

Non parametric methods, unlike parametric methods make no assumptions about the shape/functional form of  $f$ . Instead the approach is to estimate  $f$  that gets us as close to the data points as possible without capturing too much of the noise in the system (James et al., 2013). With parametric methods, it is possible that the functional form used to estimate the underlying function  $f$  (i.e  $f_0$ ) is substantially different from the true underlying  $f$ ; leading to a model which does not accurately fit the data. In contrast, non-parametric do not suffer from this issue since they essentially make no assumption about the underlying function  $f$ . However, non-parametric methods do suffer from high variance since it does not curtail the number of parameters used to make the fit.

Parametric time series analysis-when the underlying models are correctly specified-can provide a powerful array of tools for data analytics (Fan and Yao, 2008). Notwithstanding, any parametric models are at best only an approximation to the true stochastic dynamics that generates a given data set. Meaning, parametric methods are plagued with the issues of models biases. According to Fan et al. (2003), "Many data in applications exhibit nonlinear features such as nonnormality, asymmetric cycles, bimodality, nonlinearity between lagged variables, and heteroscedasticity." They require nonlinear models to describe the law that generates the data. However, beyond the linear time series models, there are infinitely many nonlinear forms that can be explored. This would be an undue task for any time series analysts to try one model after another. A natural alternative is to use nonparametric methods. Non-parametric methods are better at reducing the possible modeling biases that plague their parametric counterparts.

### 3.2. Wolf Creek Turbines Example

This example shows how parametric methods are used within the systems model to characterize the availability of the turbines at Wolf Creek GS. The reliable performance of generating unit depends on its availability on demand. Grid demands on the dam facility as a whole determines what capacity the combined turbines must be operating at. The Wolf Creek dam generates power using 6 Francis type turbines. If the demand on the system require all units to be working at full capacity, then all 6 turbines will have to be available on demand.

Availability in reliability terms, has to do with two separate events—failure and repair. Therefore, assigning confidence levels to values of availability cannot be done parametrically, and a technique such as Monte Carlo simulation is employed, based upon the estimated values of the parameters of time-to-failure and time-to-repair distributions. Passing the flow through the turbines of a hydroelectric generating station requires that the generating equipment is available and that the generated power can be accepted by the grid. If



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the grid is unavailable then turbines can still be operated with a significantly reduced discharge capacity under commonly called “spill-no load condition”. Modeling of the availability of turbines to pass the flow can be modeled as a homogeneous or non-homogeneous Poisson Process. Homogeneous approach was applied to estimate the generating equipment unavailability. Extensive data was available for all generating stations in the Wolf Creek Dam System.

Analysis results for the Wolf Creek GS are provided below. Wolf Creek GS has six generating units with the following characteristics.

Unit numbers	Speed-no load discharge (m <sup>3</sup> /s)	Discharge per unit m <sup>3</sup> /s
		Max
1,2,3,4,5,6	25	142

When a unit is unavailable, the total discharge capacity through that turbine is only 25.0 m<sup>3</sup>/s. At full capacity, the discharge capacity through turbines equals 142 m<sup>3</sup>/s each. Occurrence of failures follows a homogeneous Poisson process with mean time between failures (MTBF) having the exponential distribution with the parameter  $\lambda = 24.18$  days. These parameters were determined by fitting parametric distributions to the failure data with the best performing (fitting) distribution chosen to characterize the failure of the turbine units. The duration of the failures; which is a sum of the time to repair and the actual repair duration, is characterized by a lognormal distribution as shown below.

Duration of failures follows the lognormal distribution with the following parameters:

Mean:  $\mu=27.72$  hours

Standard deviation:  $\sigma=189.4$

Location parameter:  $\gamma=0.1435$

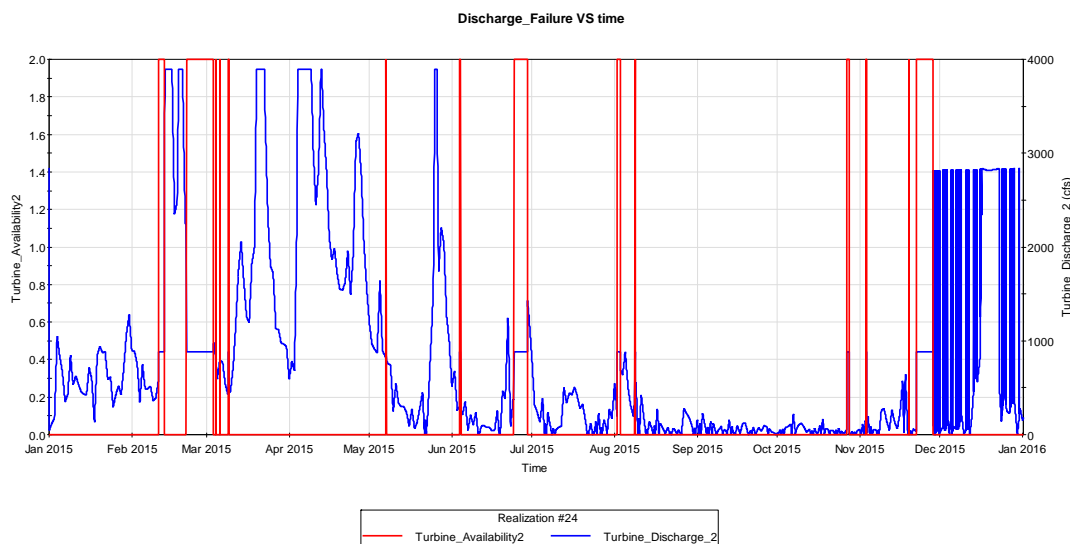


Figure 3: plot of Discharge and Availability as a function of time



Figure 2 is a plot from the preliminary model test run and shows the availability of the turbine and discharge through the turbine as a function of time. The rate of occurrence of the turbine failures are independent and identically distributed in time and thus a homogeneous Poisson process. The failure durations however, are characterized by a lognormal. It can be observed from the plot-especially the failures in the months of March, June and December-that the Unit 2 turbine has a constant rate discharge of 25 cubic meters. The rest of the time when the Turbine is available, the amount of discharge through the turbine are dictated by the SEPA curve in combination with the upstream daily flow. If Head elevation exceeds the SEPA prescribed maximum, then the turbines will be operated at capacity. On the other hand if the Head water elevation is below the prescribed Minimum SEPA level, then the turbines will be shut to conserve head.

#### **4. MODEL (STRUCTURAL) UNCERTAINTY**

Models/Simulators do not perfectly characterize a system, thus no matter how well a model is constructed, there will always be some discrepancy between the system and the simulator. Inescapably, there will be simplifications in the physics, based on features that are too complicated to be included in the model, features omitted due to lack of knowledge, disparities between the scales on which the model and the system operate, and simplifications and approximations in solving the mathematical equations underlying the system (Vernon et al., 2010). Thus, understanding structural uncertainty is one of the most challenging aspects of the uncertainty analysis.

Another challenging aspect of structural uncertainty is quantifying the uncertainty that arises due to the parametrization of only the salient aspects of the system; thus, resulting in unmodelled physical processes. In model development, certain physical processes will inevitably be neglected if there's a belief that these processes have little to no effect on the models accuracy yet adds complexity to the mathematical description. Moreover, during model development, there may be a failure to include certain physical processes due to lack of knowledge about those processes.

##### **4.1. Input Parameter Uncertainty**

Models of natural systems are made up of parameters that quantify physical processes and properties. These parameters must accurately characterize how the system properties affect system behavior. Our knowledge of the suitable values of these input parameters is often incomplete or based on limited experimental investigations (Woodhouse et al., 2015). If the underlying physics of a system is misrepresented, then the meaning of the model and the interpretation of the parameters will be called into question (Vernon et al., 2010).

This example is taken from another similar project-Mattagami Basin-Hydroelectric complex. The example here demonstrates how the input uncertainty in daily temperature is modeled using historical monthly mean (normal), warmest and coldest temperature data. A plot of the historical data can be seen in figure 3. The warmest and coldest temperatures

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for the day were taken to be two standard deviations from the mean of a normal distribution. Thus a truncated normal distribution was fit to the historical data to account for the uncertainty in the daily temperature. Figure 4 shows a monte-carlo simulation output plot for a 12 month period. From the plot, the variation in the daily inputs demonstrates that the normal distribution is being sampled on a daily basis to calculate the daily temperature based on the parameters of the normal distribution for that month. Of course, this is a simple approximation to capture the daily fluctuations in temperature. This model-although simple-does a good job at capturing the uncertainty in the input parameter (average monthly temperature) for calculating the average daily temperature.

### 4.2. Accounting for Input parameter Uncertainty: Mattagami Basin Example

This example is taken from another similar project-Mattagami Basin-Hydroelectric complex. The example here demonstrates how the input uncertainty in daily temperature is modeled using historical monthly mean (normal), warmest and coldest temperature data. A plot of the historical data can be seen in figure 3. The warmest and coldest temperatures for the day were taken to be two standard deviations from the mean of a normal distribution. Thus a truncated normal distribution was fit to the historical data to account for the uncertainty in the daily temperature. Figure 4 shows a monte-carlo simulation output plot for a 12 month period. From the plot, the variation in the daily inputs demonstrates that the normal distribution is being sampled on a daily basis to calculate the daily temperature based on the parameters of the normal distribution for that month. Of course, this is a simple approximation to capture the daily fluctuations in temperature. This model-although simple-does a good job at capturing the uncertainty in the input parameter (average monthly temperature) for calculating the average daily temperature.

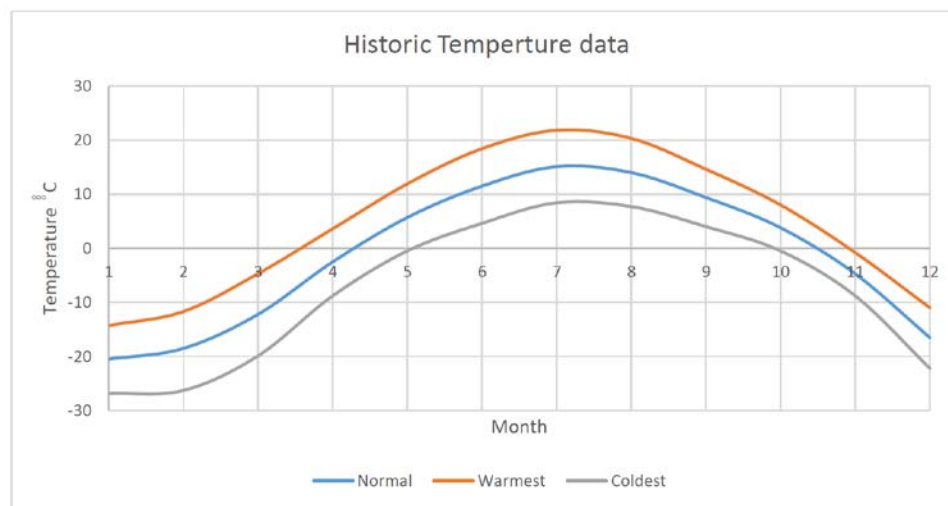
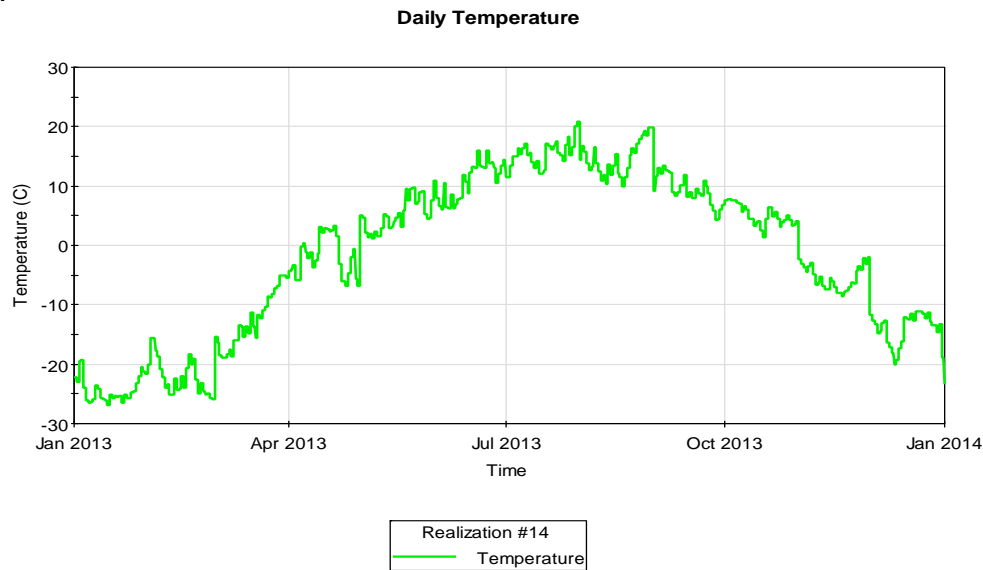


Figure 4: Plot Historical Temperature data for Mattagami Basin



*Figure 5: Plot of Simulated daily temperature values for a year*

## 5. OBSERVATIONAL ERROR

Observational uncertainty arises due to errors in the measurement of natural systems, resulting discrepancies between the real system observations and the outputs produced by the simulator. According to Woodhouse et al. (2015), “The aleatory aspect of many natural systems precludes a precise measurement”. Lack of measurement precision also adds to observational uncertainty.

For example, let’s take hydrological modeling for instance; direct measurements of rainfall runoff with stream gauge sensors will inevitably have errors associated with the accuracy of the sensors. Furthermore, many observations of natural systems are not direct and rely on models to relate a direct measurement to a quantity of interest. Taking turbine discharge measurements in hydropower generation as an example, rating tables are usually used to compute the discharge through the turbines as a function of head water elevation and the opening of the turbine sluices. These indirect observations will additionally propagate errors due to the epistemic uncertainty in the models they adopt. These are similar to structural uncertainty the only difference being that these are data collected by the dam operators and not generated in the model. Thus, we have no control over these observational errors and no arrangements have yet been made to directly account for the effect of these errors. In the end, we expect the sensitivity analysis of the model parameters to account for any uncertainty propagated due to observational errors in the input data used for the modeling.

## 6. SCENARIO UNCERTAINTY

A scenario is a plausible description of how a system might evolve over time but absent particular probabilities. Scenario uncertainty is akin to sensitivity analysis. The basic concept is that a set of scenarios is identified based on possible combinations of input data. The output of the model for each scenario of inputs is determined and evaluated against the solutions for other scenarios. This provides an insight into the consequences of each scenario.

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However, since no probabilities are associated with the respective input, correspondingly there are no probabilities, relative or absolute, for the various scenario outputs. It may not be possible to estimate the probability of one particular outcome and thus scenarios of outcomes are sometimes relied upon. A scenario is a plausible description of how a system might evolve over time but absent particular probabilities. Scenario uncertainty is akin to sensitivity analysis. The basic concept is that a set of scenarios is identified based on possible combinations of input data. The output of the model for each scenario of inputs is determined and evaluated against the solutions for other scenarios. This provides an insight into the consequences of each scenario. However, since no probabilities are associated with the respective input, correspondingly there are no probabilities, relative or absolute, for the various scenario outputs.

The scenario example in this case is taken from the Mattagami project example where there was the need to construct different scenarios of the model with different electrical backup arrangements. Table 1 shows the alternative scenarios for electrical backup arrangements. All the other aspects of the systems model remain unchanged with the only difference in each scenario model being the spillway gates electrical back-up arrangements. The idea here is to run each scenario for a very long period-in this case a 1000 years- and determine which alternative arrangement offers the safest alternative from a dam breach perspective. Of course further sensitivity of each alternative may be performed and the results aggregated over several runs to accurately arrive at the optimal electrical back-up configuration. A similar electrical back-up scenario analysis is one of the objectives of the wolf Creek GS project study.

Alternatives	Electrical Arrangement Description	No. of breaches in 1000 years	Magnitude of largest breach(breach elevation>199.3m)
Alternative A	Power From Grid Only	49	202.57m
Alternative B	2 Large Diesel Generators in parallel	24	200.11m
Alternative C	1 medium Size Diesel Generator per gate	21	200.00m
Alternative D	Grid Supplying Main Power with a Large Size Generator on Standby	21	200.58m
Alternative E	Grid Supplying Main Power with 2 Large Size Generators on Standby	12	200.34m

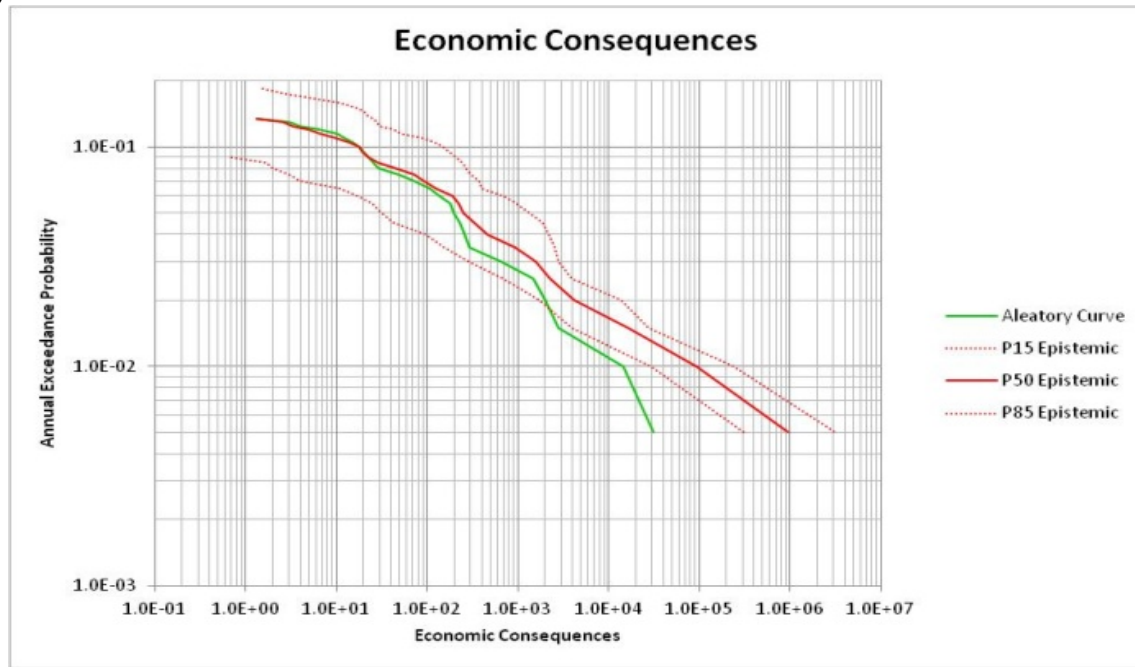
*Table 3.Electrical Configuration Alternatives*



Figure 6: Electrical Configuration Alternatives

## 7. ACCOUNTING FOR UNCERTAINTY IN FORECASTS OR PREDICTIONS: STAMFORD EXAMPLE

The Stamford Hurricane Protection Barrier levee system model outputs both the range of F-N curves generated during the epistemic loop as well as both the aleatory and epistemic (with percentiles) of the F-N curves (Figure 9). The Logic Tree (model simulator) permits the generation of uncertainty in the simulation parameters itself and to carry them through the calculation of aleatory variability. Practically, that means that after running one simulation and calculating one mean and one set of critical percentiles, another simulation process is run, generating another mean and another set of percentiles (very similar to the first one), and a third, fourth, or nth simulation is run, thus generating a new stochastic layer around the results. The correlations caused by common epistemic uncertainties are automatically carried through the simulations to the final results. The simulator runs thousands of Monte Carlo simulations in order to find the P10, P50 and P90 epistemic percentiles as well as the aleatory curve of the uncertainty in the outputs generated (see figure 8).



*Figure 7. Aleatory and Epistemic Uncertainty Curves for Levee System*

## 8. CONCLUDING REMARKS

The problems arising from the characterization of uncertainty in quantitative models for physical systems are exceedingly common across different domains. This involves a substantial uncertainty quantification task. The first step in finding a solution to this problem is being able to identify the varying forms of aleatory and epistemic uncertainties pertaining to the complex system being modeled. Subsequently, it is imperative to develop a framework within which to characterize the uncertainty about the complex systems. This framework is vital to unifying all of the sources of aleatory and epistemic uncertainties. Within this framework, all of the scientific, technical, computational, statistical and environmental issues can be addressed in principle and then characterized using the appropriate statistical methods. A model validation process is also needed to calibrate the model. Such validation process must provide a more unified approach to analyzing the several aspects of the overall model and the associated discrepancy between the model and the underlying system. This is typically achieved by comparing model evaluations (predictions) with real world system data/observations.

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