Strategic Decision-Making for Supply Chain Design and Expansion: the Case of Drinking Water and Irrigation Systems

Hakob Avetisyan[1], Mirosław J. Skibniewski[2],
Abhinandh Ramasami[1], Niranjan Mahamuni[1]

[1] Department of Civil and Environmental Engineering, E-209, 800 N. State College Blvd, California State University Fullerton, CA 92834, havetisyan@fullerton.edu

[2] Center of Excellence in Project Management, Department of Civil and Environmental Engineering, 1188 Glenn L. Martin Hall, University of Maryland, College Park, MD 20742, mirek@umd.edu

Abstract

Decision-making for supply chains in any industry providing products and services for profit is multi-dimensional. Traditionally, decisions made were based on the principle of meeting the demand. While meeting the demand is of highest priority, other important aspects include the cost, policy and regulatory compliance, and competition in the market. To arrive at an outcome that will satisfy all these aspects collectively and arrive at an equilibrium from which no participant in the market wants to deviate can be challenging. The problem becomes even more complex when there are more than one non-cooperative suppliers in the market and the decision-maker for a larger or lead supplier should decide the best strategic move supporting supply network expansion given the anticipated moves of other smaller or follower suppliers in the market. To aid decision-making in this process, a two-level leader-follower problem known as "Stackelberg game" is developed, in which the lower-level problem solves an equilibrium problem, which when combined with upper-level problem becomes a mathematical problem with equilibrium constraints (MPEC). Stackleberg games are commonly used by governments for analyzing regulations on the economy as a whole or on particular industry. Stackelberg game is regarded as a non-cooperative game where the follower makes its move by accepting the leader's choice and the leader, by anticipating that the follower makes its choice, solves for both the upper and lower-level problem variables in order to maximize its own profit or for any other chosen.

Introduction

A supply chain is one of the most important aspects for running any business successfully, and if done strategically and properly it can benefit the companies in a long-run. In order to be able to analyze the potential of improvements it is worth to define what is exactly the supply chain and how can one improve the processes of decision-making. There are many definitions in the literature for supply chain management (SCM). Since past few decades the supply chain management is growing rapidly and many researchers were targeting its different angles with a hope that the new findings will allow to serve the industries to be more stable and cost effective. It is about the

reliability and cost effectiveness when it comes to decision-making while meeting the expected quality. By expected quality it is referring to the acceptable and agreed conditions of anything that the client is looking for in any industry. In some cases the quality can be considered the least satisfactory level of functionality or specifications of delivered goods and products, while in other cases the level of the satisfaction will be exceeded to secure future business with the same client or others. In any case, the decision-making process is not straightforward and requires more advanced analysis that can allow evaluation of actions to be taken.

Definitions and the Problem

Supply chain management was defined over the last few decades, modified and improved along with technological advancement in the industry. Since the 1990's, one of the early definitions of SCM was by Novak and Simco (1991), which in particular defined it as the flow of goods from supplier that gets the goods from the manufacturer and distributor who is responsible for delivering the goods to the final user. Likewise, in 1993 Cooper and Ellram (1993) suggested that SCM is an integrative philosophy for managing the total flow of goods through distribution channel from the supplier to the final user. Yet, in 2001 Chopra and Meindl (2001) defined it as a chain that consists of all involved stages that are involved directly or indirectly in the process of fulfilling a customer request. In a similar approach, Mentzer et al. (2001) presented it as "The systemic, strategic coordination of the traditional business functions and the tactics across these business functions within a particular company and across businesses within the supply chain, for the purposes of improving the long-term performance of the individual companies and the supply chain as a whole." Given these definitions it can be acknowledged that over time indeed all researchers agree that this process is not straightforward and requires a careful and integrated analysis of the processes and steps involved.

One option to address the problem of making informed decisions is to look at many representatives in the market collectively yet each and each of them will have its own goals and perspective. Such understanding will help in forming the market structure in a game-theoretic setup where those who represent the market would be the players. In real life there will be larger players who will be treated as leaders as their decisions will drive the market movement while other may be seen as followers.

Traditionally, all the decisions-made by the suppliers were based on the principle of meeting the demand, while the profit was the target element for any company. Over time, due to the changing nature of the market policies, regulations and rules where suppliers operate and need to comply, the single goal of just meeting the demand while maximizing the profits is becoming more challenging. As such other equally important aspects include the cost reductions due to increasing worldwide competition, policy and regulation compliance. To find an outcome that will satisfy all these aspects together and arrive to an equilibrium from where no player in the market wants to deviate can be problematic. This market is not a cooperative setup and each player has its own objectives of surviving in the market and expanding its business by expanding its supply capacities. While for the follower companies it might be more challenging to invest in expansion

of their market the large companies can allow themselves to face such expenses and yet in the long-run be even more profitable.

To aid decision-makers in the process of SCM and strategic planning a mathematical game-theoretic bi-level leader-follower problem known as Stackelberg game is developed, in which the lower-level problem formulated for this research solves an equilibrium problem, which when combined with upper-level problem is known as mathematical problems with equilibrium constraints (MPEC). Stackleberg games are commonly used by governments for analyzing regulations on an economy or on a particular industry. Stackelberg game is thought to be a non-cooperative game where the follower makes its move by accepting leader's choice and the leader by anticipating that follower takes its choice solves its problem for both upper and lower-level problem variables for own profit maximization or for any other objective. The problem becomes even more challenging if the market consists of more than one leader in the upper level. For simplicity we will target the case when one large supplier acts as a leader on the top level problem for profit maximization. To illustrate the idea of a supply network, one can consider the water supply where the product is being delivered to the final user through the system where the water can be supplied from different wells or water treatment plants. It is considered that water treatment plants and others seek profit maximization and the more they produce and supply the better they will be. There is one specific difference in the water supply network, where the water is delivered to the final user through the pipelines and if the leader expands the capacities then the followers may use it, given the fees to be paid for operating it, which can be seen as an expense the followers may face in any other industry setup for utilizing any of other avenues for their business. To make the model more detailed and realistic it is also important to consider the environmental impact in terms of cost and controlled carbon market, which is done in the developed model below.

Formulation

The generalized formulation of bi-level problems is given as (Bard, 1998):

$$\min_{x \in X} F(x, y)$$
$$\text{s.t.}$$
$$G(x, y) \leq 0$$
$$\min_{y \in Y} f(x, y)$$
$$\text{s.t.}$$
$$g(x, y) \leq 0$$
$$x, y \geq 0$$

From the structure of bi-level problem we notice that the upper-level player (the leader) solves the problem for x and the lower-level player (the follower) solves for y. In a case of game-theoretic approach this can be thought of as a Stackelberg game. If draw parallels between the Stackleberg game and zero-sum games we can state that in two-person zero-sum game the gain of one player is equal to the loss of the other player. In the Stackelberg game this condition does not hold, since

whatever is gained by one player is not equal to other player's loss. This idea should not be confused with the gain and loss of potential payoff from the market while players compete with each other. Also, in zero sum-game players move simultaneously while in the case of Stackelberg game leader makes the first move then gets information from the lower-level player behavior and adjusts its move accordingly. In both problems there are also similarities, which are perfect information about each other's strategy and resulting payoffs, and non-cooperativeness. This process continues as long as it takes to determine the most favorable solution for a leader (Fricke, 2003).

In the lower-level problem there is a Nash-Cournot competition, which means that an equilibrium problem needs to be solved. This problem is solved as complementarity problem by applying KKT conditions. Complementarity problem is defined as finding a vector ($z \in R^n$) which satisfies the following conditions or to show that the vector z does not exists (Cottle, Pang, & Stone, 2009):

$$z \geq 0$$
$$q + Mz \geq 0$$
$$z^T \cdot (q + Mz) = 0$$

where vector $q \in R^n$ and M is a matrix $\in R^n$.

To illustrate the structure of KKT conditions the lower-level problem will be considered separately. Also, for completeness of KKT conditions, an equality constraint is added to the above presented lower-level problem. In bi-level problems the variable associated with upper-level problem is considered exogenous for the lower-level problem. This is due to the fact that leader makes its move first and the follower takes it as given, which in economic terms is a price taker. So, here in the lower-level problem all players are price takers. Hence, by considering x as given value (x') for a follower and converting the problem to maximization as in the developed model we get the following lower-level problem with its KKT conditions:

$$\max_{y \in Y} f(x', y)$$
$$\text{s.t.}$$
$$g(x', y) \leq 0$$
$$h(x', y) = 0$$
$$y \geq 0$$

Corresponding first order KKT conditions for constrained maximization problem would be:

$$y: \quad \frac{\partial f}{\partial y} - \lambda \frac{\partial h}{\partial y} - \mu \frac{\partial g}{\partial y} \leq 0; \qquad y \geq 0$$
$$y \cdot \left(\frac{\partial f}{\partial y} - \lambda \frac{\partial h}{\partial y} - \mu \frac{\partial g}{\partial y} \right) = 0$$
$$\mu: \quad g(x', y) \leq 0; \quad \mu \geq 0; \quad \mu \cdot g(x', y) = 0$$
$$\lambda: \quad h(x', y) = 0$$

Equation $\mu \cdot g(x', y) = 0$ is known as generalization of complementary slackness conditions for linear programs (Winston, 2009; Gabriel & Leuthold, 2010; Hobbs, 2001).

To be able to deal with nonlinearity terms in KKT conditions we need to write all inequality constraints in a followers' problem in the form (Fricke, 2003):

$$g_i(x', y) \geq 0$$

and in this case the complementary slackness condition would be:

$$\mu_i \cdot g_i(x', y) = 0$$

which when used with disjunctive programming technique will result to the following constraints:

$$\mu_i \leq M \cdot r_i$$
$$g(x', y) \leq M(1 - r_i)$$

where r_i is a binary variable for replacing complementarity by disjunctive constraint.

     After these steps the followers' problem gets combined with the leader's problem and can be solved with known standard IP/MIP solvers, such as in GAMS or CPLEX. The schematic representation of the chain can be presented as the following Figure 1 below:
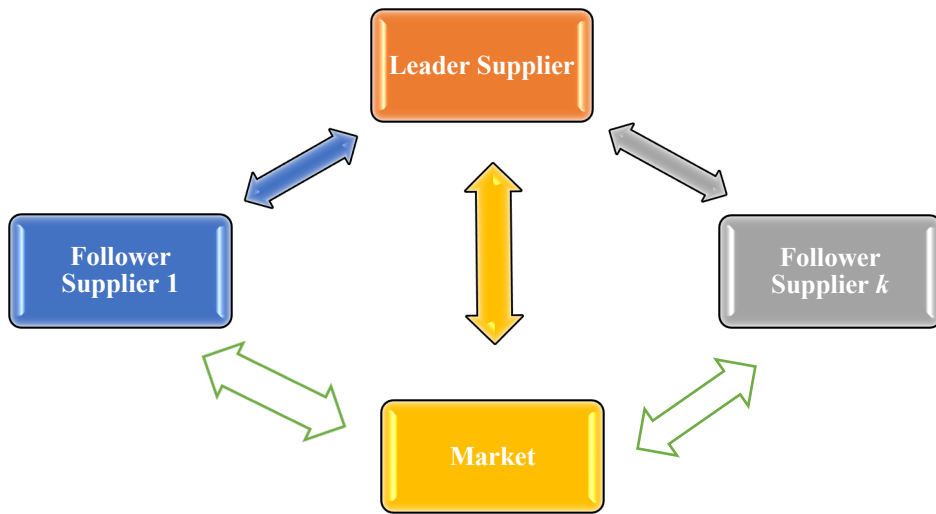

Figure1. Simplified Schematic representation of a SCM

Notation for the Model
This section presents the notation and sets used in the model formulation. Note that most of capitalized notation refers to the same parameter or variable for the leader's problem.

Indices:

        f indicates the followers
        i indicates the node of origin

j	indicates the node of destination
t	indicates discrete time period in 5 year increments for which optimized capacities need to be added

Sets:

F	set of all followers
N	set of all nodes
T	set of all time periods included in the model

Parameters:

A_{ijt}, B_{ijt}	Intercept, slope, respectively, of the linear demand functions for pipeline capacity expansion by leader between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$
a_{fijt}, b_{fijt}	Intercept, slope, respectively, of the linear demand functions for water supplied through pipelines by followers $f \in F$ between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$
U_{ijt}, W_{ijt}	Intercept, slope, respectively, of the linear demand functions for water supplied through pipelines by leader between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$
KQ_{ijt}	Fixed cost term if capacity expansion by pipeline if selected by leader between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$
$TechQ_{ijt}$	Expended pipeline operational capacities, between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$
$techq_{fijt}$	Existing pipeline operational capacities for followers $f \in F$, between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$
H_{it}	is the wellhead or plant capacity at node $i \in N$ over time $t \in T$
$CONS_{jt}$	Consumption at node at node $j \in N$ over time $t \in T$
DQ_{ijt}	Dependence factor for supplies through leader's pipeline expanded capacities between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$
dq_{fijt}	Dependence factor for supplies through follower's $f \in F$ existing pipeline between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$
ρ_{ijt}	Discounting factor, which may vary between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$

Variables:

Q_{ijt}	The amount of pipeline capacity expansion (and supply) by leader between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$
q_{fijt}	The amount of supplied water through pipeline by followers $f \in F$ between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$
XP_{ijt}	Binary variable for pipeline fixed costs if expansion is selected between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$
α_{fijt}	Shadow price of technical capacity constraint on pipelines for followers' $f \in F$ problem between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$

Functions:

$CP_{ijt}(dd)$ — Costs for pipeline capacity expansion as a function of distance and diameter between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$

$CCP_{ijt}(dd)$ — Carbon costs associated with pipeline capacity expansion as a function of distance and diameter between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$

$CQP_{ijt}(dd)$ — Costs for supplied water by leader through pipeline between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$

$cqp_{fijt}(dd)$ — Costs for supplied water by followers $f \in F$ through pipeline between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$

$ccp_{fijt}(dd)$ — Carbon costs associated with supplied water by followers $f \in F$ through pipeline between nodes $i \in N$ and $j \in N$ at time $t \in T$ and $i \neq j$

Leader in this problem is maximizing its own profits based on followers' market behavior. Its decision variables are capacity expansions as pipelines and accordingly the supply volumes. The objective function for leader's problem is given in (1):

$$
\max_{Q_{ijt}} \sum_i \sum_j \sum_t \left(\left(\left(A_{ijt} - B_{ijt} \left(\sum_f q_{fijt} + \left(Q_{ijt} - Q_{ij(t-5)} \right) \right) \right) \left(Q_{ijt} - Q_{ij(t-5)} \right) \right. \right.
$$
$$
+ \left(U_{ijt} - W_{ijt} \left(\sum_f q_{fijt} + Q_{ijt} \right) \right) Q_{ijt} - \left(CP_{ijt}(dd)\left(Q_{ijt} - Q_{ij(t-5)} \right) \right)
$$
$$
\left. - \left(CQP_{ijt}(dd)Q_{ijt} \right) - \left(CCP_{ijt}(dd)Q_{ijt} \right) - \left(KQ_{ijt} \cdot XP_{ijt} \right) \right) \rho_{ijt} \qquad (1)
$$

Leader's Problem:
The leader's problem has the following constraints, which are formulated based on technological and natural conditions of production/supply and dependency preferences of a customer from a particular supplier:

$$
0 \leq \left(Q_{ijt} - Q_{ij(t-5)} \right) \leq TechQ_{ijt} \cdot XP_{ijt}, \quad \forall i,j,t \qquad (2)
$$

$$
\sum_i \sum_j \left(\sum_f q_{fijt} + Q_{ijt} \right) \leq H_{it}, \quad \forall f,i,j,t \qquad (3)
$$

$$CONS_{jt} - \sum_i \sum_j \left(\sum_f q_{fijt} + Q_{ijt} \right) = 0, \quad \forall f, i, j, t \tag{4}$$

$$Q_{ijt} \leq DQ_{ijt} \sum_i \sum_j \left(\sum_f q_{fijt} + Q_{ijt} \right), \quad \forall f, i, j, t \tag{5}$$

$$q_{fijt} \leq dq_{fijt} \sum_i \sum_j \left(\sum_f q_{fijt} + Q_{ijt} \right), \quad \forall f, i, j, t \tag{6}$$

$$Q_{ijt} = 0, \qquad \forall i, j, t \; when \; i = j \; and \; when \; defined \; by \; user \; (i.e. political \; obstacle) \tag{7}$$

$$XP_{ijt} \in \{0,1\}, \qquad \forall i, j, t \tag{8}$$

$$Q_{ijt} \geq 0, \qquad \forall i, j, t \tag{9}$$

In leader's problem the capacity expansion is subject to technical limitations for a given time interval between origin and destination nodes. Such limitation is enforced through constraint (2) that is limiting pipeline capacity expansion in a given time interval. In addition to technological constraints the capacity of expansion can be limited because of the wellhead capacity, which is expressed through constraint (3). In this constraint players from lower level problem are also included since the capacity of a particular underground basin or source can be used by few players and hence the total capacity of expansions and accordingly flow capacities cannot exceed the maximum capacity possible to extract from a particular location. Constraint (4) is the market clearing condition which in other terms ensures that the consumption is met and there is no excess amount of water pumped and left unused. Consumers are free in their preferences and may prefer to have more capacities from one supplier and less from another supplier. Having information about consumer's preferences leader can make more reasonable decision for its investments in getting market share. Constraints (6) are designed for accommodating consumer's preferences into decision making process. In other commodity cases due to constraints (6) the decision model is also useful for consumers, who may decide for preferable conditions and strategies for them or their company and consequently develop contracts based on suggestions of the model. Constraints (7) enforce zero capacities for nodes itself, since there is no need to have capacity starting and ending at the same location. Constraints (8) are requirements for certain variables integrality and constraint (9) is non-negativity requirements.

Followers' Problem:
Followers want to maximize their profits by maximizing their production at the same time taking leader's production quantities as fixed. Followers are included in this problem with fixed capacities, which means they do not solve the problem for capacity expansion. The reasoning behind this approach is that leader would use all its options to add the gap between demand and supply if it turns to be a profitable investment to make, otherwise the dual variables included in followers' problem will provide information about the needs for followers' capacity expansions. The objective function for followers' problem is given in (i):

$$\max_{q_{fijt}} \sum_i \sum_j \sum_t \left(\left(\left(a_{fijt} - b_{fijt} \left(\sum_f q_{fijt} + Q_{ijt} \right) \right) q_{fijt} \right. \right.$$

$$\left. \left. - \left(\sum_f \left((cqp_{fijt}(dd)q_{fijt}) + (ccp_{fijt}(dd)q_{fijt}) \right) \right) \right) \rho_{ijt} \right. \qquad (i)$$

Followers have only technical limitations on their production capacities. Those are presented below:

$$0 \le q_{fijt} \le Techq_{fijt}, \quad \forall f, i, j, t \qquad\qquad (\alpha_{fijt}) \qquad (ii)$$

Constraint (ii) enforce capacity limitations, which are related to the upper levels of existing capacities in terms of pipelines.

KKT conditions for followers problem:

To formulate KKT conditions it is necessary to use derivation. To ease the situation we used a prime on the top of those terms that are associated with the term used for derivation. For instance q'_{fijt} is one of those followers in the lower level $\sum_f q_{fijt}$,

$$0 \le \left(-a'_{fijt} + 2b'_{fijt} \cdot q'_{fijt} + b'_{fijt} \left(\sum_f q_{fijt} - q'_{fijt} \right) + b'_{fijt} \cdot Q_{ijt} + cqln'_{fijt}(dd) \right.$$

$$\left. + ccp'_{fijt}(dd) \right) \rho_{ijt} + \alpha'_{fijt} \perp q'_{fijt} \ge 0, \quad \forall f, i, j, t \qquad (I)$$

$$0 \le -q'_{fijt} + Techq'_{fijt} \perp \alpha'_{fijt} \ge 0 \qquad \forall f, i, j, t \qquad (II)$$

Dual variables for non-negativity constraints are omitted. For further discussion of KKT conditions and its modifications see (Cottle, Pang, & Stone, 2009; Winston, 2009). Following the steps discussed above the KKT conditions need to be formulated as disjunctive program (not shown here).

Disjunctive form of followers' problem:

Once disjunctive program is formulated it can be combined with upper level problem and solved for equilibrium point indicating the supply volumes by both the upper and lower level players that

slows to get the best outcome in the market and plan strategically. The combined problem and the case study are not presented in this paper due to space limitations.

Conclusions:

Application of bi-level problems were proven to provide non-intuitive findings that benefit both the upper-level and lower-level players in any industry. The preliminary results of this study indicate such instances in supply chain management area as well. In previous studies authors found the importance of Stackelberg problem setup for natural gas supply network expansion between Russia and China (Avetisyan, 2013). It is expected that finding of this case study will provide similar valuable information and can be expanded and adopted by any other sector or industry's supply chain management system. The formulation in this paper is simplified to consider just few constraints for illustration perspective, but the actual larger scale model considers more than one commodity type that can be supplied by the suppliers, which gives the model more flexibility for managing the entire set of supplies.

References:

Avetisyan, H. (2013). Sustainable Infrastructure Modeling and Policy Analyses: Construction, Energy and Transportation Industries. Collections: Civil & Environmental Engineering Theses and Dissertations – University of Maryland Theses and Dissertations. URI: http://hdl.handle.net/1903/14570

Bard, J. F. (1998). *Practical bilevel optimization: algorithms and applications.* Netherlands: Kluwer Academic Publishers.

Chopra S. and Meindl's (2001), Supply Chain Management: Strategy, Planning, and Operation, Prentice Hall of India.

Cooper M. C. and L. M. Ellram, (1993). "Characteristics of Supply Chain Management and the Implications for Purchasing and Logistics Strategy," International Journal of Logistics Management, (4: 2), pp. 13-24.

Cottle, R. W., Pang, J.-S., and Stone, R. E. (2009). *The Linear Complementarity Problem.* Boston: Academic Press.

Fricke, C. (2003). *An Introduction to Bilevel Programming.* University of Melbourne, Department of Mathematics and Statistics,. Melbourne: University of Melbourne.

Gabriel, S., and Leuthold, F. (2010). Solving Discretely-Constrained MPEC Problems with Applications in Electric Power Markets. *Energy Economics , 32* (1), 3-14.

Hobbs, B. F. (2001, May 1). Linear Complementarity Models of Nash–Cournot Competition in Bilateral and POOLCO Power Markets. *IEEE TRANSACTIONS ON POWER SYSTEMS , 16* (2), pp. 194-202.

Mentzer J. T. , DeWitt V, Keebler K. S., Min S., Nix N. W. and Smith. C. D (2001). "Defining Supply Chain Management, "Journal of Business Logistics, (22: 2).

Novack R. A. and Simco S. W., (1991). "The industrial procurement process: a supply chain perspective", Journal of Business Logistics, (12: 1), pp. 145-67.

Winston, W. L. (2009). *Operations Research: Applications and Algorithms* (4th Edition ed.). Canada: Curt Hinrichs.